STOCHASTIC PROGRAMMING FOR ASSET ALLOCATION IN PENSION FUNDS

IEGOR RUDNYTSKYI
JOINT WORK WITH JOËL WAGNER

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INTRODUCTION

Common approaches for asset allocation / ALM in pension funds:

- Immunization methods
- Asset optimization
- Surplus optimization
- Liability-driven investment strategies
- Stochastic control
- Stochastic programming (SP)
- Monte-Carlo simulation methods (MC)

RESEARCH PURPOSE

- Review possible models
- Build a scalable model (in R)
- Analyze the convergence
- Analyze the sensitivity
- Compare the performance of the SP approach with MC methods

OPTIMIZATION PROBLEM

Possible objective functions:

- Maximize the total value of assets
- Maximize the expected value of the utility
- Maximize the funding ratio
- Minimize the contribution rate or the capital injection, etc.

Risk constraints:

- Chance constraints (ruin probability)
- Integrated chance constraints (TVaR)

Optimize values:

- At the final nodes
- Also at intermediate nodes

EXAMPLE OF SP

Based on J.R. Birge and F. Louveaux
Introduction to Stochastic Programming, p. 21

Problem framework:

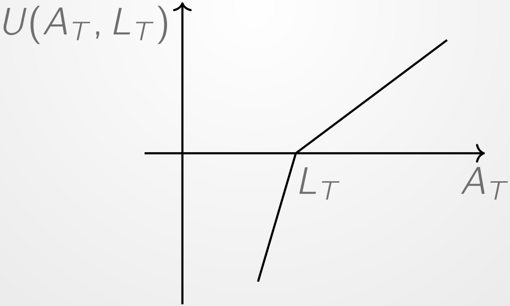
- T: planning horizon
- A_0: initial wealth (assets)
- A_T: wealth at T (depending on an economic model)
- L_T: target wealth linked to the liabilities
- Two asset classes available for investment

Problem: find the optimal asset allocation
Challenge: stochastic returns

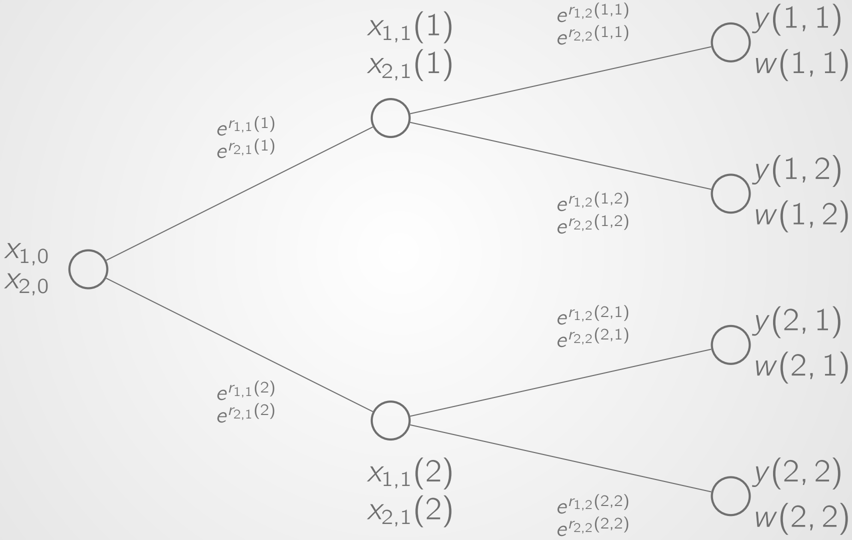
EXAMPLE OF SP (CONT'D)

(Linear) utility function:

- $U(A_T, L_T) = q \cdot (A_T - L_T)^+ - p \cdot (L_T - A_T)^+$
- q: surplus reward
- p: shortage penalty



EXAMPLE OF SCENARIO TREE



UNDERLYING ECONOMIC MODEL

Estimation of A_T:
Vector-autoregressive model (of order p in matrix form)

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \Theta_1 \boldsymbol{r}_{t-1} + \Theta_2 \boldsymbol{r}_{t-2} + ... + \Theta_p \boldsymbol{r}_{t-p} + \boldsymbol{\epsilon}_t$$

Example of VAR(1) for two assets:

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \Theta \boldsymbol{r}_{t-1} + \boldsymbol{\epsilon}_t$$

$$r_{1,t} = m_1 + \theta_{1,1} \cdot r_{1,t-1} + \theta_{1,2} \cdot r_{2,t-1} + \epsilon_{1,t}$$
$$r_{2,t} = m_2 + \theta_{2,1} \cdot r_{1,t-1} + \theta_{2,2} \cdot r_{2,t-1} + \epsilon_{2,t}$$

APPLICATION: MONTHLY RETURNS

Descriptive statistics

Returns	Bonds	Stocks
Mean	0.0034	0.0071
Std. dev.	0.0108	0.0444
Correlation matrix of returns		
Bonds	1	-0.1803
Stocks	-0.1803	1

VAR CALIBRATION

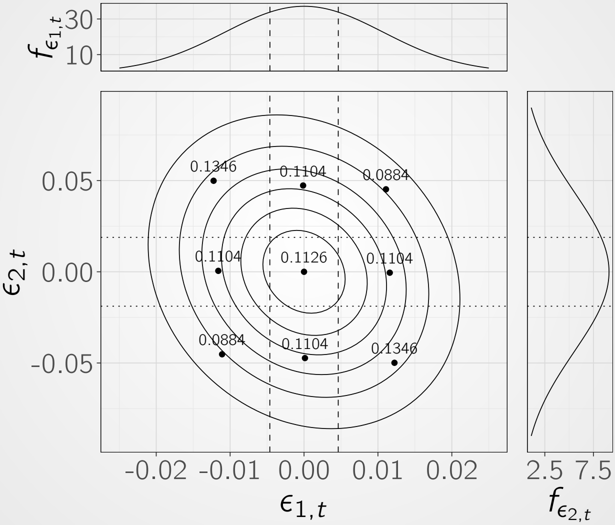
$$r_{1,t} = 0.0035 + 0.0126 \cdot r_{1,t-1} - 0.0141 \cdot r_{2,t-1} + \epsilon_{1,t}$$
$$r_{2,t} = 0.0064 - 0.1270 \cdot r_{1,t-1} + 0.1605 \cdot r_{2,t-1} + \epsilon_{2,t}$$

	$\epsilon_{1,t}$, bonds	$\epsilon_{2,t}$, stocks
$\epsilon_{1,t}$, bonds	1	-0.1743
$\epsilon_{2,t}$, stocks	-0.1743	1
Std. dev., $\sigma(\epsilon_{i,t})$	0.0107	0.0438

SCENARIO TREE GENERATION METHODS

- Sampling methods (Kouwenberg (2001))
- "Bracket-mean" and "bracket-median" (Miller III and Rice (1983), Smith (1993))
- Moment matching method via integration quadratures
  (Miller III and Rice (1983), Smith (1993))
- "Optimal discretization" (Pflug (2001))
- Other more exotic methods (e.g. Hibiki (2006))

"BRACKET-MEAN" FOR $(\epsilon_{1,t}, \epsilon_{2,t})$



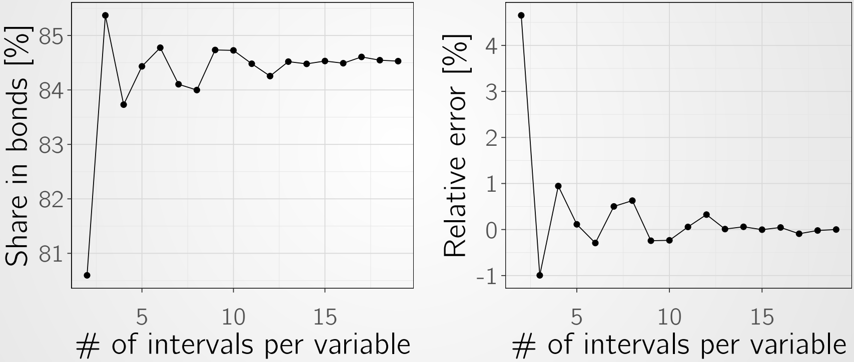
CONVERGENCE & SENSITIVITY ANALYSES

- Convergence of the optimal solution with respect to the number of outcomes
- Sensitivity of the optimal solution to changes in parameters of the model

Optimization problem:

- Utility function:
  $U(A_T, L_T) = q \cdot (A_T - L_T)^+ - p \cdot (L_T - A_T)^+$
- Optimized only at final nodes

CONVERGENCE ANALYSIS

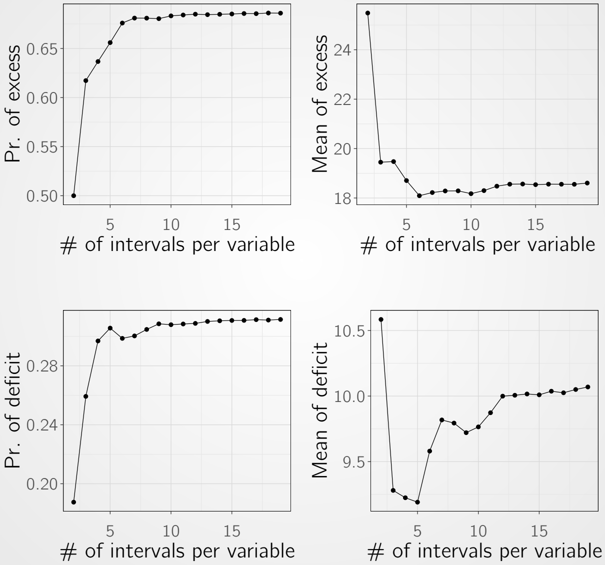


CONVERGENCE ANALYSIS (CONT'D)

Key performance indicators:

- Probability of excess: $\mathbb{P}(A_T - L_T > 0)$
- Probability of deficit: $\mathbb{P}(A_T - L_T < 0)$
- Mean of surplus given excess: $\mathbb{E}(A_T - L_T | A_T - L_T > 0)$
- Mean of shortage given deficit: $\mathbb{E}(A_T - L_T | A_T - L_T < 0)$

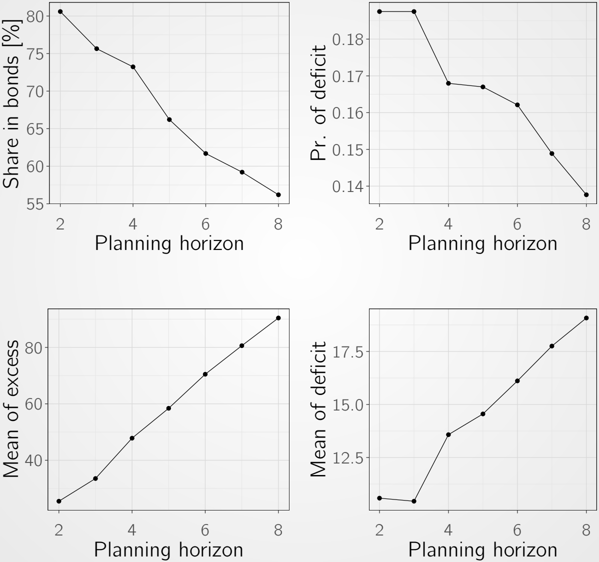
CONVERGENCE ANALYSIS (CONT'D)



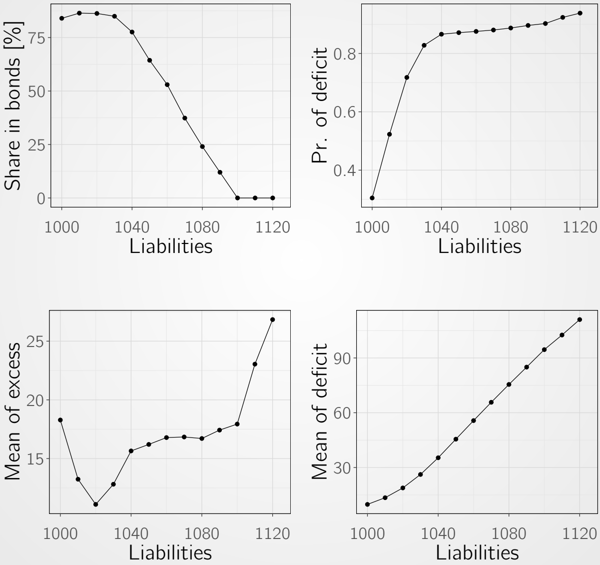
SENSITIVITY ANALYSIS

- Planning horizon T
- Target wealth L_T ($T = 2$)
- Shortage penalty p
- Bond's mean return μ_1
- Volatility of stocks' residuals $\sigma(\epsilon_{2,t})$

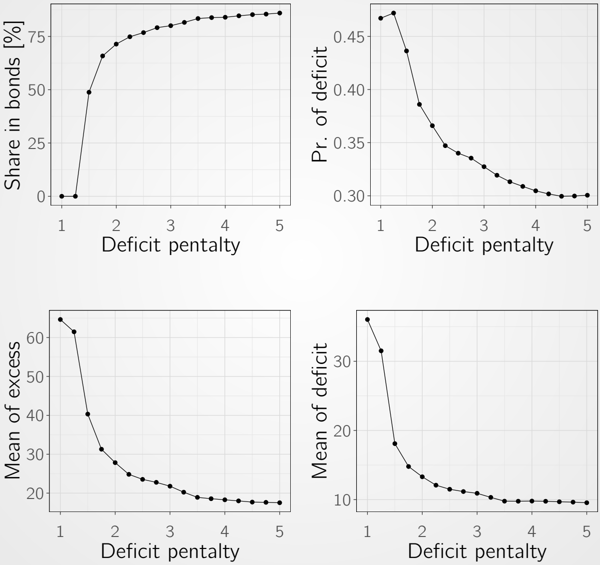
SENSITIVITY ANALYSIS: T



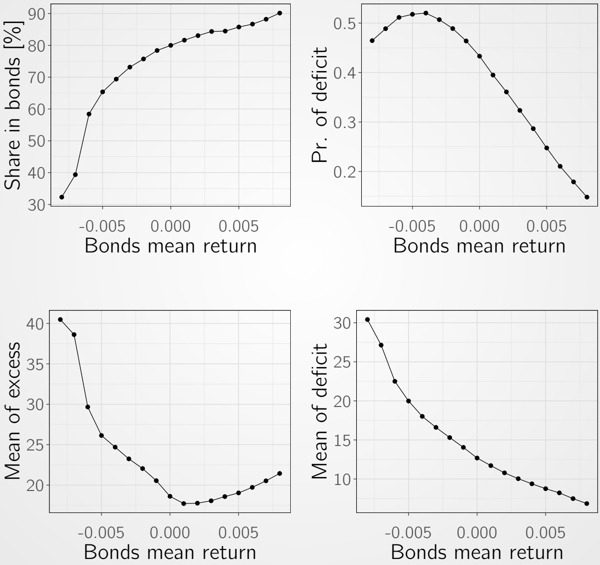
SENSITIVITY ANALYSIS: L_T ($T = 2$)



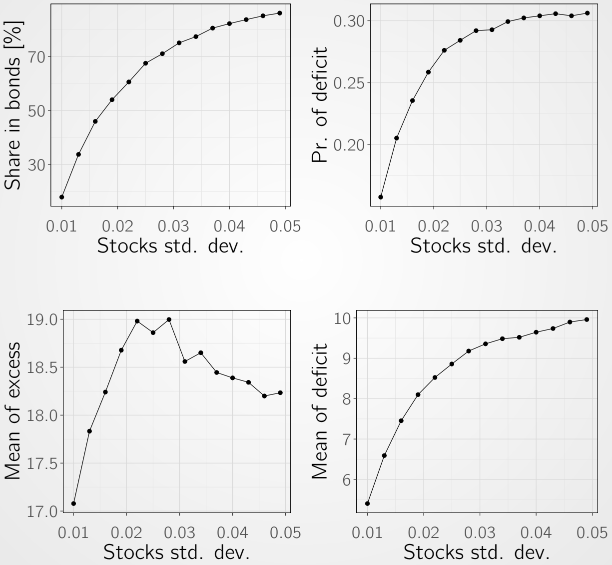
SENSITIVITY ANALYSIS: p



SENSITIVITY ANALYSIS: μ_1



SENSITIVITY ANALYSIS: $\sigma(\epsilon_{2,t})$



COMPARISON WITH MONTE CARLO

- Simulate $N = 10000$ paths of VAR model.
- Fix the initial asset allocation at $t = 0$. Using “Buy&Hold" strategy calculate the final wealth for each of the simulated path.
- Estimate quantities of interest.

RESEARCH SUMMARY

We have studied:

- Various scenario tree generation techniques
- Possible software and solvers
- The convergence of the optimal solution with respect to the bushiness of the scenario tree
- The relation between the optimal solution and model's characteristics (planning horizon T, target wealth L_T, etc)

Possible extensions:

- Use more sophisticated economic models
- Use stochastic liabilities
- Impose regulatory constraints

THANK YOU!